

On The Behavior of Strategies in Iterated Games Between Relatives

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Abstract — The theory of iterated games provide a system framework to explore the players' relationship in a long-term. In this paper we consider the iterated prisoner's dilemma game (IPD) played between relatives . Two state automata are used to play infinitely iterated the two players where each action can be mis-implemented with small error probability. the payoff matrix using the perturbation approach is computed . Using a different values of the average relatedness between players and different values of the payoff variables (R, S, T, P) , the behavior of strategies for iterated prisoner's dilemma game in each situation is studied .

Keywords : Repeated Games, Transition matrix, Finite automata, Perturbed Payoff.



1 INTRODUCTION

Game theory provides a quantitative framework for analyzing the behavior of rational players . The theory of iterated games in particular provide a system framework to explore the players' relationship in a long-term. It has been an important tool in the behavioral and biological sciences and it has been often invoked by economists, political scientists, anthropologists and other scientists who were interested in human cooperation (Axelrod 1984, Aumann 1981, Fudenberg and Mask 2007; 1990).

The prisoner's dilemma game (Rapoport and Chammah 1995) is the most famous example of iterated games . The (IPD) is now regarded as an ideal experimental platform for the evolution of cooperation among selfish players and it attracts wide interest since Robert Axelrod's IPD tournaments . However, the publication of Axelrod's book in the 1980s was largely responsible for bring this research to the attention of other areas outside of game theory, including evolutionary computation, conflict resolution, evolutionary biology, networked computer systems and promoting cooperation between opposing countries. Despite the large literature base that now exists this is an outstanding area of research.

In this paper, we study the iterated prisoner's dilemma game in which there is a relationship between the players . The average relatedness between the players is given by r , which is a number between 0 and 1. A simple way to study

games between relatives was proposed by Maynard Smith for the Hawk-Dove game (Hines and Smith 1979 ; Grafen 1979).

In iterated prisoner's dilemma, the two players have two options, either to Cooperate (C) or to defect (D). In one-shot prisoner's dilemma game ,the strategy D is the best, and it dominates the cooperative option . But if this game played repeatedly many times then the picture will change. In this situation the strategy D will not be the dominant strategy for a long time. In iterated games, the number of possible grows exponentially with the number of rounds in the game (Nowak *at el.* 1995 ; Rubinstein 1986).

We assume that ,when the players plays the iterated Prisoner's Dilemma there is some noise , i.e. In each round, a player makes a mistake with probability ϵ leading to the opposite move. Since there is a lot of strategies of (IPD) , we just consider all strategies that can be implemented by deterministic finite state automata with one or two states.

Finite state automata have been used extensively to study the iterated games . In our case, we have two states each state is labeled by C or D. In state C the player will cooperate in the next move ; in state D the player will defect. Each strategy starts in one of those two states. Each state has two outgoing transitions (either to the same or to the other state): one transition specifies what happens if the opponent has cooperated and one if the opponent has defected (Zagorsky *at el.* 2013 ; Nowak *at el.* 1995) .

In (Nowak *at el.* 1995) they studied prisoner's dilemma where they used the played repeatedly by two-state automata , they computed the 16x16 payoff matrix for limiting case of vanishingly a small noise term affecting the interaction . In this

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paper we define the transition rule of each automaton that depends on the initial state of the game and on the payoff of last move , and we compute the payoff matrix of iterated prisoner's dilemma games in which there is a relationship between the players . Then we describe the method that we shall follow to compute the 16 x16 payoff matrix for iterated prisoner's dilemma game with noise played by finite state automata. After calculating the payoff matrix we study the effect of different values of average relatedness and different values for the payoff values (R , S , T , P) on the behavior of the 16 strategies .

2 PRISONER'S DILEMMA BETWEEN RELATIVES

The Prisoner's Dilemma (PD) is a non-zero sum game formulated by the mathematician Tucker building on the ideas of Flood and Dresher in 1950 . Since then, it has been discussed extensively by game theorists, economists, mathematicians, political scientists, biologists, philosophers, ethicists, sociologists, and the computer scientists (Brunauer *at el.* 2007). Many variations of the Prisoner's Dilemma have been devised, one of them being the Iterated Prisoner's Dilemma (IPD), which is at the center of attention in this paper. In this game , the two players have two options, either to Cooperate (C) or to defect (D) and the payoff values are traditionally called T (for temptation to betray a cooperating opponent), S (for sucker's payoff when being betrayed while cooperating oneself), P (for punishment when both players betray each other), and R (for reward when both players cooperate with each other). Their values vary from formulation to formulation of the prisoner's dilemma. Nevertheless, the inequalities $S < P < R < T$ and $2R > T + S$ are always observed between them. The last one ensures that cooperating twice (2R) pays more than alternating one's own betrayal of one's partner (T) with allowing oneself to be betrayed by him or her (S) . We can represent this game by the following payoff matrix :

$$\begin{matrix} & C & D \\ C & (R & S) \\ D & (T & P) \end{matrix} \tag{1}$$

Now , we assume that this game is played between relatives. A simple way to study games between relatives was proposed by Maynard Smith for the Hawk- Dove game (Hines and Smith 1979 ; Grafen 1979). Consider a population where the average relatedness between players is given by r , which is a number between 0 and 1. There are two possible methods to study the games between relatives. The "inclusive fitness "

method adds to the payoff of a player r times the payoff to his co-player .The personal fitness method, proposed by Grafen 1979 modifies the fitness of the player by allowing for the fact that a player is more likely than other players of the population to meet co-player adopting the same strategy as himself. We regard the inclusive fitness method to study the iterated prisoner's dilemma that played by finite state automata and subjected to a small error (Hines and Smith 1979). If we assume that there is a relationship between the players , then by using the inclusive fitness method , the payoff matrix of the prisoners dilemma game is given by

$$\begin{matrix} & C & D \\ C & (R(1+r) & S+rT) \\ D & (T+rS & P(1+r)) \end{matrix} \tag{2}$$

where r is the average relatedness between players , which is a number between 0 and 1.

3 FINITE AUTOMATA AND TRANSITION RULE

The potential of the automata theory for the analysis of games was first suggested in the economics literature by Aumann (1981). Finite-state automata have been used extensively to study iterated games including prisoner's dilemma (Zagorsky *at al.* 2013) . In this paper we use an automata with two states as we mentioned that in introduction , each state of the automaton is labeled by C or D , in the state C the player will cooperated in the next move , in the state D the player will defect .All the strategy starts in one of those two states . Each state has two outgoing transition : one transition specifies what happens if the opponent has cooperated and one if the opponent defected.

There are 32 automates with two different states, but some of these automaton describe automata with the same behavior . Thus there are only 26 automata encoding unique strategies (Nowak *at el.* 1995 ; El Seidy *at el.* 2013) . These strategies include AllC , AllD , Tit-For-Tat (TFT) and Win-Stay, Lose-shift (WSLS)(Fig.1).

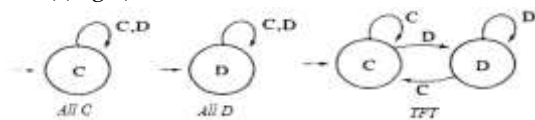


Fig.1

Each round leads to one of the four possible outcomes (C,C) , (C,D) , (D,C) or (D,D), where the first position denotes the option chosen by the player and the second that of the co-player. These outcomes , from the player's point of view, are specified by his payoff R , S , T or P, which can be numbered by 1 , 2 , 3 , 4. The 16 possible transition rules can be defined

by a quadruple (p_1, p_2, p_3, p_4) of zeros and ones. Where p_i denotes the probability to cooperated after each of the four outcomes (C,C),(C,D),(D,C) and (D,D). Thus (1, 1, 1, 1) is the rule of cooperate (AllC) and (0, 0, 0, 0) is the rule of defect (AllD), while (1, 0, 1, 0) is the rule of imitating the adversary's last move. The automata using this rule and with initial state C plays Tit-for-Tat. TFT starts in state C and subsequently does whatever the opponent did in the last round. This strategy is very successful in an error-free environment, but in a noisy environment TFT achieves a very low payoff against itself since it can only recover from a single error by another error (Zagorsky *at el.* 2013). To simplify, we label these rules by S_i where i ranges from 1 to 15 and is the integer given. Hence S_0 is AllD and S_{10} is Tit-for-Tat.

4 THE COMPETITIONS BETWEEN STRATEGIES WITH NOISE

How one rule matches against another depends on the initial condition of this rule. For example, consider the automaton with rule $S_{12} = (1, 1, 0, 0)$ against the automaton with rule $S_{14} = (1, 1, 1, 0)$, therefore:

(a) If both automaton start with C, they keep playing C forever, The sequence is:

$S_{12} : C C C C C C C C : \dots$

$S_{14} : C C C C C C C C : \dots$

(b) If both automaton start with D, they keep playing D forever, we get:

$S_{12} : D D D D D D D D : \dots$

$S_{14} : D D D D D D D D : \dots$

(c) If S_{12} starts with C and S_{14} with D, we get:

$S_{12} : C C C C C C C C : \dots$

$S_{14} : D D C C C C C C : \dots$

(d) If S_{12} start with D and S_{14} with C, the result is:

$S_{12} : D D D D D D D D : \dots$

$S_{14} : C C C C C C C C : \dots$

In the infinitely iterated game, the payoff is the average payoff per round, in our example, the player who use the transition rule S_{12} get the payoff $R(1+r)$ in cases (a) and (c), and $P(1+r)$ in case (b), and $T+rS$ in case (d).

Now, if we assume that the implementation of a move is subject to error. This means that there is a small probability $\epsilon > 0$, that one state is replaced by another. The corresponding transition rule in this case is given by a quadruple like S_i but with ϵ instead of 0 and $1-\epsilon$ instead of 1. The problem now is to compute the payoff for strategy $S_i(\epsilon)$ against $S_j(\epsilon)$. For more generally, let us consider a strategy $E = (e_1, e_2, e_3, e_4)$ and $F = (f_1,$

$f_2, f_3, f_4)$ where e_k and f_k are the probability to play C after outcome k ($k = 1, 2, 3, 4$). Therefore, we get the transition matrix between the four states R, S, T and P as shown in the stochastic matrix (3).

$$M = \begin{pmatrix} e_1 f_1 & e_1(1-f_1) & (1-e_1)f_1 & (1-e_1)(1-f_1) \\ e_2 f_3 & e_2(1-f_3) & (1-e_2)f_3 & (1-e_2)(1-f_3) \\ e_3 f_2 & e_3(1-f_2) & (1-e_3)f_2 & (1-e_3)(1-f_2) \\ e_4 f_4 & e_4(1-f_4) & (1-e_4)f_4 & (1-e_4)(1-f_4) \end{pmatrix} \quad (3)$$

(We note the interchange of 2 and 3, due to the fact that one player's S is the other player's T). If the matrix M is irreducible (as is always the case when $0 < e_k, f_k < 1$ for all k , and for particular if E and F correspond to strategies S_i), the matrix M has a unique left eigenvector $\alpha = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ to the eigenvalue 1 such that $0 < \alpha_k$ for $k = 1, 2, 3, 4$ and $\sum \alpha_k = 1$. These α_k denote the relative frequencies of the states k of the corresponding Markov chain. They specify the limit in the mean payoff for strategy E against strategy F which is $\alpha_1 R + \alpha_2 S + \alpha_3 T + \alpha_4 P$. Since our game is played between relatives as we assumed, therefore the payoff will be $\alpha_1 R(1+r) + \alpha_2(S+rT) + \alpha_3(T+rS) + \alpha_4 P(1+r)$. the F player's payoff is obtained by interchanging α_2 and α_3 .

Now, for any noise level $\epsilon > 0$, we can compute the payoff obtained by the automaton using transition rule S_i against the automaton using transition rule S_j using the following approach: (we will exemplify it for S_{12} against S_{14}). The four possible initial condition leads to three possible stationary states $R_1; R_2$, and R_3 , where R_1 denotes the run where the both player use C. while R_2 is the run where the S_{12} -player plays D and S_{14} -player plays C and R_3 is the run where the both player use D. Now, suppose we are in regime R_1 . A rare perturbation cause S_{12} -player to play D, this leads to regime R_2 with probability $1/2$. Suppose that the perturbation happened in regime R_2 . With probability $1/2$, it cases the S_{12} -player to switch from D to C. If this happens while S_{14} -player plays C, we are in regime R_1 , but if S_{14} -player plays D, this leads to regime R_3 . Suppose now that the perturbation occurs in regime R_3 , with probability $1/2$, it cases the S_{12} -player to switch from D to C if this happens while S_{14} -player plays D, we are in regime R_1 suppose now that the perturbation affects the S_{14} -player, He plays C instead of D, while S_{12} -player plays D, this leads to regime R_2 . Thus the perturbation of R_1 leads with probability $1/2$ to R_2 , and the perturbation of R_2 leads with probability $1/2$ to R_1 and with probability $1/2$ to R_3 , while R_3 leads with probability $1/2$ to R_1 and with probability $1/2$ to R_2 . The corresponding transition matrix is:

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \quad (4)$$

and the corresponding stationary distribution is (1/2 , 1/3 , 1/6).Therefore ,the S_{12} -player receives the average payoff $1/2R(1+r) + 1/3(T+rS) + 1/6P(1+r)$ per round. This method , repeated for each of the 256 entries, leads to the 16x16 payoff matrix (Table 2).Now, we said that a strategy S_i is out competed by strategy S_j if the following conditions are satisfied : $V(S_j, S_i) \geq V(S_i, S_i)$ and $V(S_j, S_j) \geq V(S_i, S_j)$. For example, the payoff matrix between the S_{12} -player and S_{14} -player ,after substituting with Axelrod's payoff values ($R = 3, S = 0, T = 5, P = 1$) and assume that the average relatedness between the players is 0.5 , is given by :

$$\begin{matrix} & S_{12} & S_{14} \\ S_{12} & (3.38 & 4.17) \\ S_{14} & (3.33 & 4.5) \end{matrix} \quad (5)$$

Then , we notice that $V(S_{14}, S_{14}) > V(S_{12}, S_{14})$, but $V(S_{14}, S_{12}) < V(S_{12}, S_{12})$, therefore the strategy S_{12} is not out competed by the strategy S_{14} .

5 THE EFFICIENCY OF AVERAGE RELATEDNESS ON THE BEHAVIOR OF STRATEGIES

In this section we study the behavior of strategies and the effect of the average relatedness (r) between players on cooperative , defective and other behaviors in competition between strategies . we use different values for r and different values for the variables of payoff values (R, S, T, P). In Table 1 we summarize all strategies that out compete the strategy S_i in each situation that we studied and we see that :

(a)For the payoff values ($R = 3, S = 0, T = 5, P = 1$) and for $r = 0.0001$ and $r = 0.999$, we get from tables (1), (3) and (4) : If the average relatedness between players was small, such that $r = 0.0001$, then we see that all strategies are out competed by at least two other strategies . also we note that the strategy S_0 (AllD) can defeat the greatest number of strategies (exactly 12), while the strategy S_6 (idiot strategy) cannot defeat any other strategy . Strategies S_{14} and S_{15} are the weakest strategies , they out competed by exactly 11 strategies . here , the cooperative strategies are invaded by a defective strategies. In this case we note that the strategy S_9 (called Win-Stay Lose-Shift or Pavlov) is out competed by three strategies , which mean whenever the average of relatedness is low , then the strategy S_9 (and the other cooperative strategies) is invaded by other strategies. The strategy TFT or S_{10} make a good work and defeat the defective strategies like Grim (S_8) , AllD(S_0),and S_1 . If the

average relatedness between players was large , such that $r = 0.999$, then we see that all strategies except S_9, S_{14} and S_{15} are out competed by at least three other strategies .the strategies S_9, S_{14} and S_{15} are defeat all other strategies while the strategy S_6 cannot defeat any other strategy . Also when the average relatedness between players is high then the strategies $S_0 = (0, 0, 0, 0)$ (a defective strategy) and $S_8 = (1, 0, 0, 0)$ (a retaliator who never relents after a defection) are out competed by the largest number of strategies (exactly by 11 strategies). The ordered paired of equilibrium strategies in both cases are (S_0, S_8) , (S_3, S_{12}) , (S_5, S_{10}) and (S_{14}, S_{15}).

	(R=3,T=5,S=0,P=1)		(R=0,T=1,S=1,P=10)		(R=3,T=5,S=0,P=3)	
	r=0.0001	r=0.999	r=0.0001	r=0.999	r=0.0001	r=0.999
S_0	2,8,10	1,2,3,4,8,9,10,11,12,13,14,15	2,3,4,9,10,11,12,14	1,2,3,4,9,10,11,12,13,14,15	-----	-----
S_1	0,3,4,8,10	3,5,9,10,11,13,14,15	3,5,9,10,11	3,5,9,10,11,12,13,14,15	2	2
S_2	1,9,10,11	1,5,9,10,11,12,13,14,15	1,5,9,10,11,14	1,5,9,10,11,13,14,15	0,6,7,8,14,15	0,6,7,8,13,14,15
S_3	0,4,8,9,11,12	9,11,12,13,14,15	9,11,14	9,11,13,14,15	1,6,7,9,14,15	1,6,7,9,13,14,15
S_4	0,8	1,3,9,11,12,13,14,15	9,12,14	3,9,12,13,14,15	0,6,8	0,3,8,11
S_5	0,1,2,8,9,10	9,10,13,14,15	9	9,13,14,15	1,6,7,9	1,6,7,9,10
S_6	0,1,2,3,4,5,7,8,9,10,11,12	2,3,4,5,9,10,11,12,13,14,15	2,3,4,5,10,11,12	2,3,5,9,10,11,12,13,14,15	-----	-----
S_7	0,1,2,3,4,5,9,12	3,5,9,11,13,14,15	3,5,11	9,10,11,12,13,14,15	-----	-----
S_8	0,2,9,10	0,1,2,3,4,9,10,11,12,13,14,15	2,3,4,9,10,11,12	2,3,4,9,10,11,12,13,14,15	1	-----
S_9	0,1,8	-----	-----	-----	-----	-----
S_{10}	5,9,11,14,15	5,9,11,14,15	9,11,13,14,15	9,11,13,14,15	0,6,8,9	0,6,8,9,14,15
S_{11}	0,1,4,5,8,9,12,14,15	9,14,15	13,14,15	-----	1,9,14,15	1,6,9,14,15
S_{12}	0,1,2,3,4,8,9	3,9,11,13,14,15	9	9,11,13,14,15	6,8,9,14,15	6,8,9,14,15
S_{13}	0,1,2,3,5,8,9,12	3,9,14,15	9	-----	9,14,15	9
S_{14}	0,1,2,3,4,5,7,8,9,12,13,15	15	9,13	-----	-----	-----
S_{15}	0,1,2,3,4,5,7,8,9,12,13,14	14	9,13	-----	-----	-----

Table 1: the strategies that out compete the strategy S_i ($i = 0, 1, \dots, 15$) with different values of R, S, T, P and r .

(b) If the payoff values were as follow ($R = 0, S = 1, T = 1, P = 10$) (called the chicken game) and for $r = 0.0001$ and 0.999 , from tables (1) , (5) and (6) : for $r = 0.0001$, the defective strategies show some activity and trying to avoid invasion by other strategies , but they cannot avoid TFT and WLS strategies. The strategy WLS is the strongest strategy and no other strategy can defeat it .Some strategies like S_5 are ambitious and try to invade other strategies . The ordered paired of equilibrium strategies are (S_0, S_8) , (S_3, S_{12}) , (S_5, S_{10}) , (S_9, S_{11}) , (S_{11}, S_{13}) , (S_{11}, S_{14}) , (S_{11}, S_{15}) and (S_{14}, S_{15}). If we assume that $r = 0.999$ the cooperative strategies S_{13}, S_{14}, S_{15} and the WLS(S_9) or Pavlov strategy are dominating other strategies and no other strategy can defeat any one of them . In this case the defective strategies as GRIM (S_8) and AllD are invaded by cooperative

strategies and then the cooperative behavior will be evolve between the players. The ordered paired of equilibrium strategies are (S_0, S_8) , (S_3, S_{12}) , (S_5, S_{10}) , (S_9, S_{11}) , (S_9, S_{13}) , (S_9, S_{14}) , (S_9, S_{15}) , (S_{11}, S_{13}) , (S_{11}, S_{14}) , (S_{11}, S_{15}) , (S_{13}, S_{14}) , (S_{13}, S_{15}) and (S_{14}, S_{15}) .

(c) In which the payoff values were $(R = 5, S = 0, T = 1, P = 3)$ and $r = (0.0001 \text{ and } 0.999)$, therefore from tables (1), (7) and (8) we see that when the average relatedness between players is small, such that $r = 0.0001$, we see that there is a strong competition between defective and cooperative strategies. some strategies such as WLSL, AllC and AllD. no other strategy can out compete them, however if the average relatedness between players was large, such that $r = 0.999$, the defective strategies like Grim(S_8) become stronger and no other strategy can defeat it. Some strategies are not affected by the values of r such as WLSL or Pavlov, AllC and AllD and other strategies. In two cases the ordered paired of equilibrium strategies are (S_0, S_8) , (S_3, S_{12}) , (S_5, S_{10}) , and (S_{14}, S_{15}) .

CONCLUSION

We have studied in this paper the iterated games played by finite state automata. We consider a relationship between the players who plays this game, this relationship is given by an average relatedness parameters r where $0 \leq r \leq 1$. We assumed that the automata are subjected to some small error, this error due to implementation of what the other player does. We computed the 16×16 -payoff matrix of iterated prisoner's dilemma game between relatives with noise which played by finite state automata. We studied this game with different values of R , S , T and P and different values of average relatedness between players.

For the payoff values $(R = 3, S = 0, T = 5, P = 1)$ and for $r = 0.999$ and $r = 0.0001$ we concluded that, all strategies are out competed by at least two other strategies except for, the strategy WLSL(S_9) or Pavlov if $r=0.999$ there is no strategy can defeat this strategy. Also the strategy S_6 does not affected by the relatedness average, it is a weak strategy in both cases. We saw that whenever there is a large degree of kinship, the cooperation evolve between the players, and the cooperative strategies are dominate. while if there is a small degree of kinship, the defective strategies will dominate and the defective behavior between players will evolve. If we change the order of payoff values such that $(R = 0, S = 1, T = 1, P = 10)$ (for the chicken game), we found that whenever a small degree of kinship between players, the defective strategies

show some activity and trying to avoid invasion by other strategies. In this case the strategy WLSL (S_9) or Pavlov is the strongest strategy and doesn't affected by the values of r . In case that $R > P > T > S$ and such that $(R = 5, S = 0, T = 1, P = 3)$, there is a strong competition between defective and cooperative strategies. Almost all the strategies are not significantly affected by degree of kinship between the two players. the strategy TFT(S_{10}) for large degree of kinship between players is not successful and exposed to invasion by AllD, WLSL or Pavlov, Grim, AllC and other strategies. here the direct reciprocity behavior is not the best respond for each players.

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Table 1 . the payoff matrix of repeated prisoner’s dilemma between relatives with error in implementation

	S_0	S_1	S_2	S_3	S_4	S_5
S_0	$P(1+r)$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$P(1+r)$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$\frac{T+2P}{3} + \frac{r(S+2P)}{3}$	$T+rS$
S_1	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+P}{2}(1+r)$	$\frac{S+T+P}{3}(1+r)$	$\frac{R+P}{2}(1+r)$	$\frac{2S+T+2P}{5} + \frac{r(2T+S+2P)}{5}$	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$
S_2	$P(1+r)$	$\frac{R+P}{2}(1+r)$	$\frac{S+T+2P}{4} + \frac{r(T+S+2P)}{4}$	$\frac{S+T}{2}(1+r)$	$P(1+r)$	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$
S_3	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+P}{2}(1+r)$	$\frac{S+T}{2}(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+P}{2}(1+r)$
S_4	$\frac{S+2P}{3} + \frac{r(T+2P)}{3}$	$\frac{S+2T+2P}{5} + \frac{r(T+2S+2P)}{5}$	$P(1+r)$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$\frac{S+T+2P}{4} + \frac{r(T+S+2P)}{4}$	$T+rS$
S_5	$T+rS$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{R+P}{2}(1+r)$	$T+rS$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_6	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$T+rS$	$P(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{S+2P}{3} + \frac{r(T+2P)}{3}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_7	$T+rS$	$\frac{R+2S+P}{4} + \frac{r(R+2T+P)}{4}$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{R+P}{2}(1+r)$	$T+rS$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$
S_8	$P(1+r)$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$P(1+r)$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$\frac{T+2P}{3} + \frac{r(S+2P)}{3}$	$T+rS$
S_9	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{S+T+P}{3}(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{2S+T+2P}{5} + \frac{r(2T+S+2P)}{5}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_{10}	$P(1+r)$	$\frac{S+T+P}{3}(1+r)$	$\frac{S+T+P}{3}(1+r)$	$\frac{S+T}{2}(1+r)$	$P(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_{11}	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{S+T}{2}(1+r)$	$\frac{S+T}{2}(1+r)$	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$
S_{12}	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{2S+T+P}{4} + \frac{r(2T+S+P)}{4}$	$\frac{R+S+2P}{4} + \frac{r(R+T+2P)}{4}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$(3S+T+P)/6+r*(3T+S+P)/6$	$\frac{S+T}{2}(1+r)$
S_{13}	$T+rS$	$T+rS$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$T+rS$	$T+rS$
S_{14}	$\frac{2S+P}{3} + \frac{r(2T+P)}{3}$	$T+rS$	$\frac{2R+2S+P}{5} + \frac{r(2R+2T+P)}{5}$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{2S+P}{3} + \frac{r(2T+P)}{3}$	$T+rS$
S_{15}	$T+rS$	$T+rS$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$T+rS$	$T+rS$

	S_6	S_7	S_8	S_9	S_{10}
S_0	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$T+rS$	$P(1+r)$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$P(1+r)$
S_1	$T+rS$	$\frac{R+2T+P}{4} + \frac{r(R+2S+P)}{4}$	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$	$\frac{S+T+P}{3}(1+r)$
S_2	$P(1+r)$	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$	$P(1+r)$	$\frac{S+T+P}{3}(1+r)$	$\frac{S+T+P}{3}(1+r)$
S_3	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+P}{2}(1+r)$	$\frac{S+P}{2} + \frac{r(T+P)}{2}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{S+T}{2}(1+r)$
S_4	$\frac{T+2P}{3} + \frac{r(S+2P)}{3}$	$T+rS$	$\frac{S+2P}{3} + \frac{r(T+2P)}{3}$	$\frac{S+2T+2P}{5} + \frac{r(T+2S+2P)}{5}$	$P(1+r)$
S_5	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$	$T+rS$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_6	$P(1+r)$	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{2S+P}{3} + \frac{r(2T+P)}{3}$	$T+rS$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_7	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{R+P}{2}(1+r)$	$T+rS$	$T+rS$	$\frac{R+S+T}{3}(1+r)$
S_8	$\frac{2T+P}{3} + \frac{r(2S+P)}{3}$	$T+rS$	$P(1+r)$	$\frac{R+2T+2P}{5} + \frac{r(R+2S+2P)}{5}$	$P(1+r)$
S_9	$T+rS$	$T+rS$	$\frac{R+2T+2P}{5} + \frac{r(R+2S+2P)}{5}$	$R(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_{10}	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+S+T}{3}(1+r)$	$P(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$
S_{11}	$\frac{R+S+T}{3}(1+r)$	$\frac{R+S+T}{3}(1+r)$	$\frac{R+2S+2P}{5} + \frac{r(R+2T+2P)}{5}$	$R(1+r)$	$\frac{R+S+T}{3}(1+r)$
S_{12}	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+S+2T}{4} + \frac{r(R+T+2S)}{4}$	$\frac{R+2S+3P}{6} + \frac{r(R+2T+3P)}{6}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+P}{2}(1+r)$
S_{13}	$\frac{2R+2S+P}{5} + \frac{r(2R+2T+P)}{5}$	$\frac{2R+2S+P}{5} + \frac{r(2R+2T+P)}{5}$	$\frac{R+2S}{3} + \frac{r(R+2T)}{3}$	$\frac{2R+S}{3} + \frac{r(2R+T)}{3}$	$R(1+r)$
S_{14}	$\frac{2R+2S+P}{5} + \frac{r(2R+2T+P)}{5}$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{R+2S+P}{4} + \frac{r(R+2T+P)}{4}$	$\frac{R+2S}{3} + \frac{r(R+2T)}{3}$	$R(1+r)$
S_{15}	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{R+2S}{3} + \frac{r(R+2T)}{3}$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$R(1+r)$

	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$T+rS$	$\frac{2T+P}{3} + \frac{r(2S+P)}{3}$	$T+rS$
S_1	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$	$\frac{S+2T+P}{4} + \frac{r(T+2S+P)}{4}$	$T+rS$	$T+rS$	$T+rS$
S_2	$\frac{S+T}{2}(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$	$\frac{2R+2T+P}{5} + \frac{r(2R+2S+P)}{5}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$
S_3	$\frac{S+T}{2}(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$
S_4	$\frac{T+P}{2} + \frac{r(S+P)}{2}$	$\frac{S+3T+2P}{6} + \frac{r(T+3S+2P)}{6}$	$T+rS$	$\frac{2T+P}{3} + \frac{r(2S+P)}{3}$	$T+rS$
S_5	$\frac{R+T+P}{3} + \frac{r(R+S+P)}{3}$	$\frac{S+T}{2}(1+r)$	$T+rS$	$T+rS$	$T+rS$
S_6	$\frac{R+S+P}{3} + \frac{r(R+T+P)}{3}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{2R+S+2T}{5} + \frac{r(2R+T+2S)}{5}$	$\frac{2R+2T+P}{5} + \frac{r(2R+2S+P)}{5}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$
S_7	$\frac{R+S+T}{3}(1+r)$	$\frac{R+2S+T}{4} + \frac{r(R+2T+S)}{4}$	$\frac{2R+S+2T}{5} + \frac{r(2R+T+2S)}{5}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$
S_8	$\frac{R+2T+2P}{5} + \frac{r(R+2S+2P)}{5}$	$\frac{R+2T+3P}{6} + \frac{r(R+2S+3P)}{6}$	$\frac{R+2T}{3} + \frac{r(R+2S)}{3}$	$\frac{R+2T}{3} + \frac{r(R+2S)}{3}$	$\frac{R+2T}{3} + \frac{r(R+2S)}{3}$
S_9	$R(1+r)$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{2R+T}{3} + \frac{r(2R+S)}{3}$	$\frac{R+2T}{3} + \frac{r(R+2S)}{3}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$
S_{10}	$\frac{R+S+T}{3}(1+r)$	$\frac{R+P}{2}(1+r)$	$R(1+r)$	$R(1+r)$	$R(1+r)$
S_{11}	$\frac{2R+S+T}{4}(1+r)$	$\frac{2R+S+P}{4} + \frac{r(2R+T+P)}{4}$	$R(1+r)$	$R(1+r)$	$R(1+r)$
S_{12}	$\frac{2R+T+P}{4} + \frac{r(2R+S+P)}{4}$	$\frac{R+S+T+P}{4} + \frac{r(R+S+T+P)}{4}$	$\frac{2R+S+3T}{6} + \frac{r(2R+T+3S)}{6}$	$\frac{3R+2T+P}{6} + \frac{r(3R+2S+P)}{6}$	$\frac{R+T}{2} + \frac{r(R+S)}{2}$
S_{13}	$R(1+r)$	$\frac{2R+3S+T}{6} + \frac{r(2R+3T+S)}{6}$	$\frac{2R+S+T}{4}(1+r)$	$\frac{2R+T}{3} + \frac{r(2R+S)}{3}$	$\frac{2R+T}{3} + \frac{r(2R+S)}{3}$
S_{14}	$R(1+r)$	$\frac{3R+2S+P}{6} + \frac{r(3R+2T+P)}{6}$	$\frac{2R+S}{3} + \frac{r(2R+T)}{3}$	$R(1+r)$	$R(1+r)$
S_{15}	$R(1+r)$	$\frac{R+S}{2} + \frac{r(R+T)}{2}$	$\frac{2R+S}{3} + \frac{r(2R+T)}{3}$	$R(1+r)$	$R(1+r)$

Table 2 . the payoff matrix of repeated prisoner’s dilemma between relatives with error in implementation with Axelrod values ($R=3,T=5,S=0,P=1$) and $r=0.0001$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	1.00	3.00	1.00	3.00	2.33	5.00	3.00	5.00	1.00	3.00	1.00	3.00	3.00	5.00	3.67	5.00
S_1	0.50	2.00	2.00	2.00	1.40	3.00	5.00	3.50	0.50	3.00	2.00	3.00	2.75	5.00	5.00	5.00
S_2	1.00	2.00	1.75	2.50	1.00	3.00	1.00	3.00	1.00	2.00	2.00	2.50	2.50	4.00	3.40	4.00
S_3	0.50	2.00	2.50	2.25	0.50	2.00	2.25	2.00	0.50	2.25	2.50	2.50	2.25	4.00	4.00	4.00
S_4	0.67	2.40	1.00	3.00	1.75	5.00	2.33	5.00	0.67	2.40	1.00	3.00	2.83	5.00	3.67	5.00
S_5	5.00	1.33	1.33	2.00	5.00	2.25	2.25	3.00	5.00	2.25	2.25	3.00	2.50	5.00	5.00	5.00
S_6	0.50	5.00	1.00	2.25	0.67	2.25	1.00	1.33	0.33	5.00	2.25	1.33	2.25	3.20	3.40	4.00
S_7	5.00	1.00	1.33	2.00	5.00	1.33	1.33	2.00	5.00	5.00	2.67	2.67	2.00	3.20	4.00	4.00
S_8	1.00	3.00	1.00	3.00	2.33	5.00	3.67	5.00	1.00	3.00	1.00	3.00	2.67	4.33	4.33	4.33
S_9	0.50	1.33	2.00	2.25	1.40	2.25	5.00	5.00	1.00	3.00	2.25	3.00	2.25	3.67	4.33	4.00
S_{10}	1.00	2.00	2.00	2.50	1.00	2.25	2.25	2.67	1.00	2.25	2.25	2.67	2.00	3.00	3.00	3.00
S_{11}	0.50	1.33	2.50	2.50	0.50	1.33	2.67	2.67	1.00	3.00	2.67	2.75	1.75	3.00	3.00	3.00
S_{12}	0.50	1.50	1.25	2.25	1.00	2.50	2.25	3.25	1.00	2.25	2.00	3.00	2.25	3.50	3.33	4.00
S_{13}	5.00	5.00	1.50	1.50	5.00	5.00	2.20	2.20	1.00	2.00	3.00	3.00	1.83	2.75	3.67	3.67
S_{14}	0.33	5.00	1.40	1.50	0.33	5.00	1.40	1.50	1.00	1.00	3.00	3.00	1.67	2.00	3.00	3.00
S_{15}	5.00	5.00	1.50	1.50	5.00	5.00	1.50	1.50	1.00	1.50	3.00	3.00	1.50	2.00	3.00	3.00

Table 3 . the payoff matrix of repeated prisoner’s dilemma between relatives with error in implementation with Axelrod values ($R=3,T=5,S=0,P=1$) and $r=0.999$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	2.00	3.50	2.00	3.50	3.00	5.00	3.50	5.00	2.00	3.50	2.00	3.50	3.50	5.00	4.00	5.00
S_1	3.50	4.00	4.00	4.00	3.80	4.33	5.00	4.50	3.50	4.33	4.00	4.33	4.25	5.00	5.00	5.00
S_2	2.00	4.00	3.25	5.00	2.00	4.33	2.00	4.33	2.00	4.00	4.00	5.00	3.75	5.50	4.80	5.50
S_3	3.50	4.00	5.00	4.50	3.50	4.00	4.50	4.00	3.50	4.50	5.00	5.00	4.50	5.50	5.50	5.50
S_4	3.00	3.80	2.00	3.50	3.50	5.00	3.00	5.00	3.00	3.80	2.00	3.50	4.00	5.00	4.00	5.00
S_5	5.00	4.33	4.33	4.00	5.00	4.50	4.50	4.33	5.00	4.50	4.50	4.33	5.00	5.00	5.00	5.00
S_6	3.50	5.00	2.00	4.50	2.66	4.50	2.00	4.33	4.00	5.00	4.50	4.33	4.50	5.40	4.80	5.50
S_7	5.00	4.50	4.33	4.00	5.00	4.33	4.33	4.00	5.00	5.00	5.33	5.33	5.25	5.40	5.50	5.50
S_8	2.00	3.50	2.00	3.50	3.00	5.00	4.00	5.00	2.00	4.00	2.00	4.00	3.67	5.33	5.33	5.33
S_9	3.50	4.33	4.00	4.50	3.80	4.50	5.00	5.00	4.00	6.00	4.50	6.00	4.50	5.66	5.33	5.50
S_{10}	2.00	4.00	4.00	5.00	2.00	4.50	4.50	5.33	2.00	4.50	4.50	5.33	4.00	6.00	6.00	6.00
S_{11}	3.50	4.33	5.00	5.00	3.50	4.33	5.33	5.33	4.00	6.00	5.33	5.50	4.75	6.00	6.00	6.00
S_{12}	3.50	4.25	3.75	4.50	3.66	5.00	4.50	5.25	3.66	4.50	4.00	4.75	4.50	5.33	5.00	5.50
S_{13}	5.00	5.00	5.50	5.50	5.00	5.00	5.40	5.40	5.33	5.66	6.00	6.00	5.33	5.50	5.66	5.66
S_{14}	4.00	5.00	4.80	5.50	4.00	5.00	4.80	5.50	4.50	5.33	6.00	6.00	5.00	5.66	6.00	6.00
S_{15}	5.00	5.00	5.50	5.50	5.00	5.00	5.50	5.50	5.33	5.50	6.00	6.00	5.50	5.66	6.00	6.00

Table 4 . the payoff matrix of repeated prisoner's dilemma between relatives with error in implementation with Axelrod values ($R=0, T=1, S=-1, P=-10$) and $r=0.0001$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	-10.00	-4.50	-10.00	-4.50	-6.33	1.00	-4.50	1.00	-10.00	-4.50	-10.00	-4.50	-4.50	1.00	-2.67	1.00
S_1	-5.50	-5.00	-3.33	-5.00	-4.20	-3.00	1.00	-2.00	-5.50	-3.00	-3.33	-3.00	-2.25	1.00	1.00	1.00
S_2	-10.00	-5.00	-5.00	0.00	-10.00	-3.00	-10.00	-3.00	-10.00	-3.33	-3.33	0.00	-4.75	0.50	-1.60	0.50
S_3	-5.50	-5.00	0.00	-2.50	-5.50	-5.00	-2.50	-5.00	-5.50	-2.50	0.00	0.00	-2.50	0.50	0.50	0.50
S_4	-7.00	-3.80	-10.00	-4.50	-5.00	1.00	-6.33	1.00	-7.00	-3.80	-10.00	-4.50	-3.00	1.00	-2.67	1.00
S_5	1.00	-3.67	-3.67	-5.00	1.00	-2.50	-2.50	-3.00	1.00	-2.50	-2.50	-3.00	0.00	1.00	1.00	1.00
S_6	-5.50	1.00	-10.00	-2.50	-7.00	-2.50	-10.00	-3.67	-4.00	1.00	-2.50	-3.67	-2.50	0.20	-1.60	0.50
S_7	1.00	-3.00	-3.67	-5.00	1.00	-3.67	-3.67	-5.00	1.00	1.00	0.00	0.00	-0.25	0.20	0.50	0.50
S_8	-10.00	-4.50	-10.00	-4.50	-6.33	1.00	-2.67	1.00	-10.00	-3.60	-10.00	-3.60	-4.67	0.67	0.67	0.67
S_9	-5.50	-3.67	-3.33	-2.50	-4.20	-2.50	1.00	1.00	-4.40	0.00	-2.50	0.00	-2.50	0.33	0.67	0.50
S_{10}	-10.00	-3.33	-3.33	0.00	-10.00	-2.50	-2.50	0.00	-10.00	-2.50	-2.50	0.00	-5.00	0.00	0.00	0.00
S_{11}	-5.50	-3.67	0.00	0.00	-5.50	-3.67	0.00	0.00	-4.40	0.00	0.00	0.00	-2.75	0.00	0.00	0.00
S_{12}	-5.50	-2.75	-5.25	-2.50	-2.00	0.00	-2.50	0.25	-5.33	-2.50	-5.00	-2.25	-2.50	0.33	-1.33	0.50
S_{13}	1.00	1.00	-0.50	-0.50	1.00	1.00	-0.20	-0.20	-0.67	-0.33	0.00	0.00	-0.33	0.00	0.33	0.33
S_{14}	-4.00	1.00	-2.40	-0.50	-4.00	1.00	-2.40	-0.50	-3.00	-0.67	0.00	0.00	-2.00	-0.33	0.00	0.00
S_{15}	1.00	1.00	-0.50	-0.50	1.00	1.00	-0.50	-0.50	-0.67	-0.50	0.00	0.00	-0.50	-0.33	0.00	0.00

Table 5 . the payoff matrix of repeated prisoner's dilemma between relatives with error in implementation with Axelrod values ($R=0, T=1, S=-1, P=-10$) and $r=0.999$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	-19.99	-9.99	-19.99	-9.99	-13.33	0.00	-9.99	0.00	-19.99	-9.99	-19.99	-9.99	-9.99	0.00	-6.66	0.00
S_1	-10.00	-10.00	-6.66	-10.00	-8.00	-6.66	0.00	-5.00	-10.00	-6.66	-6.66	-6.66	-5.00	0.00	0.00	0.00
S_2	-19.99	-10.00	-7.50	0.00	-19.99	-6.66	-19.99	-6.66	-19.99	-6.66	-6.66	0.00	-9.99	0.00	-4.00	0.00
S_3	-10.00	-10.00	0.00	-5.00	-10.00	-10.00	-5.00	-10.00	-10.00	-5.00	0.00	0.00	-5.00	0.00	0.00	0.00
S_4	-13.33	-8.00	-19.99	-9.99	-10.00	0.00	-13.33	0.00	-13.33	-8.00	-19.99	-9.99	-6.66	0.00	-6.66	0.00
S_5	0.00	-6.66	-6.66	-10.00	0.00	-5.00	-5.00	-6.66	0.00	-5.00	-5.00	-6.66	0.00	0.00	0.00	0.00
S_6	-10.00	0.00	-19.99	-5.00	-9.99	-5.00	-19.99	-6.66	-6.66	0.00	-5.00	-6.66	-5.00	0.00	-4.00	0.00
S_7	0.00	-5.00	-6.66	-10.00	0.00	-6.66	-6.66	-10.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S_8	-19.99	-9.99	-19.99	-9.99	-13.33	0.00	-6.66	0.00	-19.99	-8.00	-19.99	-8.00	-9.99	0.00	0.00	0.00
S_9	-10.00	-6.66	-6.66	-5.00	-8.00	-5.00	0.00	0.00	-8.00	0.00	-5.00	0.00	-5.00	0.00	0.00	0.00
S_{10}	-19.99	-6.66	-6.66	0.00	-19.99	-5.00	-5.00	0.00	-19.99	-5.00	-5.00	0.00	-10.00	0.00	0.00	0.00
S_{11}	-10.00	-6.66	0.00	0.00	-10.00	-6.66	0.00	0.00	-8.00	0.00	0.00	0.00	-5.00	0.00	0.00	0.00
S_{12}	-10.00	-5.00	-10.00	-5.00	-3.33	0.00	-5.00	0.00	-10.00	-5.00	-10.00	-5.00	-5.00	0.00	-3.33	0.00
S_{13}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
S_{14}	-6.66	0.00	-4.00	0.00	-6.66	0.00	-4.00	0.00	-5.00	0.00	0.00	0.00	-3.33	0.00	0.00	0.00
S_{15}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 6 . the payoff matrix of repeated prisoner's dilemma between relatives with error in implementation with Axelrod values ($R=5, T=1, S=0, P=3$) and $r=0.0001$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	3.00	2.00	3.00	2.00	2.33	1.00	2.00	1.00	3.00	2.00	3.00	2.00	2.00	1.00	1.67	1.00
S_1	1.50	4.00	1.33	4.00	1.40	3.00	1.00	2.50	1.50	3.00	1.33	3.00	1.25	1.00	1.00	1.00
S_2	3.00	4.00	1.75	0.50	3.00	3.00	3.00	3.00	3.00	1.33	1.33	0.50	3.00	3.00	3.00	3.00
S_3	1.50	4.00	0.50	2.25	1.50	4.00	2.25	4.00	1.50	2.25	0.50	0.50	2.25	3.00	3.00	3.00
S_4	2.00	1.60	3.00	2.00	1.75	1.00	2.33	1.00	2.00	1.60	3.00	2.00	1.50	1.00	1.67	1.00
S_5	1.00	2.67	2.67	4.00	1.00	2.25	2.25	3.00	1.00	2.25	2.25	3.00	0.50	1.00	1.00	1.00
S_6	1.50	1.00	3.00	2.25	2.00	2.25	3.00	2.67	1.00	1.00	2.25	2.67	2.25	2.40	3.00	3.00
S_7	1.00	2.00	2.67	4.00	1.00	2.67	2.67	4.00	1.00	1.00	2.00	2.00	1.50	2.40	3.00	3.00
S_8	3.00	2.00	3.00	2.00	2.33	1.00	1.67	1.00	3.00	2.60	3.00	2.60	2.67	2.33	2.33	2.33
S_9	1.50	2.67	1.33	2.25	1.40	2.25	1.00	1.00	2.20	5.00	2.25	5.00	2.25	3.67	2.33	3.00
S_{10}	3.00	1.33	1.33	0.50	3.00	2.25	2.25	2.00	3.00	2.25	2.25	2.00	4.00	5.00	5.00	5.00
S_{11}	1.50	2.67	0.50	0.50	1.50	2.67	2.00	2.00	2.20	5.00	2.00	2.75	3.25	5.00	5.00	5.00
S_{12}	1.50	1.00	2.75	2.25	0.67	0.50	2.25	1.75	2.33	2.25	4.00	3.50	2.25	2.17	3.33	3.00
S_{13}	1.00	1.00	2.50	2.50	1.00	1.00	2.20	2.20	1.67	3.33	5.00	5.00	1.83	2.75	3.67	3.67
S_{14}	1.00	1.00	2.60	2.50	1.00	1.00	2.60	2.50	2.00	1.67	5.00	5.00	3.00	3.33	5.00	5.00
S_{15}	1.00	1.00	2.50	2.50	1.00	1.00	2.50	2.50	1.67	2.50	5.00	5.00	2.50	3.33	5.00	5.00

Table 7 . the payoff matrix of repeated prisoner's dilemma between relatives with error in implementation with Axelrod values ($R=5, T=1, S=0, P=3$) and $r=0.999$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}
S_0	6.00	3.50	6.00	3.50	4.33	1.00	3.50	1.00	6.00	3.50	6.00	3.50	3.50	1.00	2.67	1.00
S_1	3.50	8.00	2.67	8.00	3.00	5.66	1.00	4.50	3.50	5.66	2.67	5.66	2.25	1.00	1.00	1.00
S_2	6.00	8.00	2.75	1.00	6.00	5.66	6.00	5.66	6.00	2.67	2.67	1.00	5.75	5.50	5.60	5.50
S_3	3.50	8.00	1.00	4.50	3.50	8.00	4.50	8.00	3.50	4.50	1.00	1.00	4.50	5.50	5.50	5.50
S_4	4.33	3.00	6.00	3.50	3.50	1.00	4.33	1.00	4.33	3.00	6.00	3.50	2.67	1.00	2.67	1.00
S_5	1.00	5.66	5.66	8.00	1.00	4.50	4.50	5.66	1.00	4.50	4.50	5.66	1.00	1.00	1.00	1.00
S_6	3.50	1.00	6.00	4.50	3.33	4.50	6.00	5.66	2.67	1.00	4.50	5.66	4.50	4.60	5.60	5.50
S_7	1.00	4.50	5.66	8.00	1.00	5.66	5.66	8.00	1.00	1.00	4.00	4.00	3.25	4.60	5.50	5.50
S_8	6.00	3.50	6.00	3.50	4.33	1.00	2.67	1.00	6.00	4.80	6.00	4.80	5.00	4.00	4.00	4.00
S_9	3.50	5.66	2.67	4.50	3.00	4.50	1.00	1.00	4.80	10.00	4.50	10.00	4.50	7.00	4.00	5.50
S_{10}	6.00	2.67	2.67	1.00	6.00	4.50	4.50	4.00	6.00	4.50	4.50	4.00	8.00	10.00	10.00	10.00
S_{11}	3.50	5.66	1.00	1.00	3.50	5.66	4.00	4.00	4.80	10.00	4.00	5.50	6.75	10.00	10.00	10.00
S_{12}	3.50	2.25	5.75	4.50	1.67	1.00	4.50	3.25	5.00	4.50	8.00	6.75	4.50	4.00	6.33	5.50
S_{13}	1.00	1.00	5.50	5.50	1.00	1.00	4.60	4.60	4.00	7.00	10.00	10.00	4.00	5.50	7.00	7.00
S_{14}	2.67	1.00	5.60	5.50	2.67	1.00	5.60	5.50	4.50	4.00	10.00	10.00	6.33	7.00	10.00	10.00
S_{15}	1.00	1.00	5.50	5.50	1.00	1.00	5.50	5.50	4.00	5.50	10.00	10.00	5.50	7.00	10.00	10.00